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Influence Variable Static Stability on the Dynamics of Ultralong Waves in a Two-Dimensional Baroclinic Model of the Atmosphere

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A two-dimensional baroclinic model of the atmosphere adapted for description of the dynamics of ultralong waves was formulated in [1, 2]. However, the results obtained there were, strictly speaking, valid only in the special case of a neutrally stratified atmosphere in which the lapse rate $\gamma=-\partial T/\partial z$ is equal to the adiabatic lapse rate $\gamma_a = g/c_p$. In reality, however, the vertical stratification of the atmosphere is stable. The temperature of the air varies with height in rather complex fashion, with $\gamma=-\partial T/\partial z\approx{}^2/_3\gamma_a$. in the troposphere, in which most of its mass is concentrated. It would therefore be interesting to establish the significance of the temperature stratification of the atmosphere for the dynamics of ultralong waves.

We shall confine ourselves in this paper to a polytropic model of the atmosphere in which the parameter γ is, along with the weighted-average temperature of an air column and the surface pressure, an unknown function that depends on the horizontal coordinates and on time. As in [1, 2], the approximation of geostrophic motions of the second kind will be used to investigate the ultralong waves. In this case, the dynamic equation system has the form

$$\mathbf{n}\times\rho f\mathbf{v}=-\nabla p-\rho\mathbf{g},\qquad \frac{\partial\rho}{\partial t}+\operatorname{div}(\rho\mathbf{v})=0,$$
$$\rho\frac{dT}{dt}-\frac{1}{c_p}\frac{dp}{dt}=\rho Q/c_p,\qquad p=R\rho T. \tag{1}$$

Here f is the Coriolis parameter, c_p is the specific heat at constant pressure, R is the gas constant, $\mathbf{v}$ is the velocity vector, $\mathbf{n}$ is the unit vector normal to the surface of the earth, $\mathbf{g}$ is the gravity vector, p, ρ, and T are the pressure, density, and temperature fields, and Q is the heat flux divergence.

Let us investigate the adiabatic case. We introduce local Cartesian coordinates: x increases eastward, y northward, and z vertically upward. Averaging the equations of continuity, thermodynamics, and motion over the entire thickness of the atmosphere, in much the same way as was done in [3, 4], we obtain evolution equations for $\hat\rho=\int_A\rho\,dz=p_0/g$ and $\bar T=\int_A T\rho\,dz/\hat\rho$ (p_0 is the surface pressure) and a diagnostic relation connecting $\bar{\mathbf{v}}=\int_A\mathbf{v}\rho\,dz/\hat\rho$ with $\hat\rho$ and $\bar T$. The equation for γ can be obtained by averaging the dynamic equation differentiated over z, recognizing that $T=T_0(x,y,t)-\gamma(x,y,t)z$, $T_0=\bar T(1+R\gamma/g)$ (T_0 is the surface temperature). Eliminating the velocity from the resulting equations, we can finally write the system of equations

$$\frac{\partial\bar T}{\partial t}+\frac{R\bar Tk_1}{f\hat\rho}(\hat\rho,\bar T)+\frac{R^2\bar T^2k_3}{fg\hat\rho k_2}(\hat\rho,\gamma)-\frac{R^2\bar T}{fgk_2}(\bar T,\gamma)$$
$$-\frac{\beta R\bar T}{f^2\hat\rho}\left(k_3+\frac{\Gamma k_1k_4}{k_2}\right)\frac{\partial\hat\rho\bar T}{\partial x}-\frac{\Gamma\beta R\bar Tk_1k_4}{f^2k_2}\frac{\partial\bar T}{\partial x}+\frac{k_4\Gamma\beta R^2\bar T^2}{f^2gk_2^2}\frac{\partial\gamma}{\partial x}=0,$$
$$\frac{\partial\gamma}{\partial t}-\frac{R\bar Tk_1^2}{f\hat\rho k_2}(\gamma,\hat\rho)+\frac{Rk_1}{fk_2}(\bar T,\gamma)-\frac{\Gamma\beta R\bar Tk_1k_4}{f^2k_2^2}\frac{\partial\gamma}{\partial x} \tag{2}$$
$$-\frac{\Gamma\beta gk_1k_3k_4}{f^2\hat\rho k_2}\frac{\partial\hat\rho\bar T}{\partial x}+\frac{\Gamma\beta gk_1^2k_4}{f^2k_2}\frac{\partial\bar T}{\partial x}=0,$$
$$\frac{\partial\hat\rho}{\partial t}-\frac{\beta R}{f^2}\frac{\partial\hat\rho\bar T}{\partial x}=0,$$

where β is the meridional gradient of the Coriolis parameter, $\Gamma=R/c_p$. $k_1=1+R\gamma/g$, $k_2=1+2R\gamma/g$, $k_3=R\gamma/g$, $k_4=1-\gamma/\gamma_a$, $(A,B)=(\partial A/\partial x)(\partial B/\partial y)-(\partial A/\partial y)(\partial B/\partial x)$.

Let us find stationary solutions of (2). Simple manipulation yields

$$\frac{1}{\hat\rho}\frac{\partial\hat\rho\bar T}{\partial y}\left(\frac{\gamma}{\bar T}k_1\frac{\partial\bar T}{\partial x}-\frac{\partial\gamma}{\partial x}\right)+\frac{\beta\gamma_ak_4}{f}\left(k_1\frac{\partial\bar T}{\partial x}-\frac{R\bar T}{gk_2}\frac{\partial\gamma}{\partial x}\right)=0,$$
$$(\gamma,\bar T)-\frac{1}{\hat\rho}\frac{\partial\hat\rho\bar T}{\partial y}\left(\frac{\partial\gamma}{\partial x}+\frac{gk_1}{R\bar T}\frac{\partial\bar T}{\partial x}\right)=0,\qquad \frac{\partial\hat\rho\bar T}{\partial x}=0. \tag{3}$$

Expressing $\partial\gamma/\partial x$ in terms of $\partial\bar T/\partial x$ from the second equation of system (3) and substituting in the first equation, we have

$$\frac{\partial\bar T}{\partial x}\left\{\left[\frac{\gamma k_1}{\hat\rho^2}\frac{\partial\hat\rho}{\partial y}+\frac{1}{\hat\rho}\frac{\partial\gamma}{\partial y}+\frac{gk_1}{R\bar T\hat\rho^2}\frac{\partial\hat\rho\bar T}{\partial y}+\frac{\beta\gamma_ak_1k_4}{f\hat\rho k_2}\right]\frac{\partial\hat\rho\bar T}{\partial y}\right.$$
$$\left.+\frac{\beta\gamma_a\bar Tk_1k_4}{f\hat\rho}\frac{\partial\hat\rho}{\partial y}+\frac{\beta\gamma_aR\bar Tk_4}{fgk_2}\frac{\partial\gamma}{\partial y}\right\}=0. \tag{4}$$

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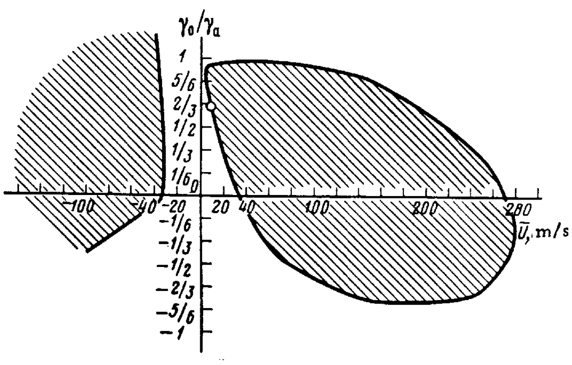

Fig. 1. Diagram of stability of zonal flow to ultralong waves as a function of average zonal flow velocity $\bar{U}_0$ and temperature stratification parameter γ_0/γ_a. The region of instability is shaded.

One of the stationary solutions is then obtained at once:* $\partial\bar{T}/\partial x=0$, $\partial\gamma/\partial x=0$, $\partial\hat{\rho}/\partial x=0$ or $\bar{T}=\bar{T}_0(y)$, $\gamma=\gamma_0(y)$, $\hat{\rho}=\hat{\rho}_0(y)$, where the function $\bar{T}_0(y)$, $\gamma_0(y)$, $\hat{\rho}_0(y)$ are arbitrary and independent.

We linearize system (2) with respect to perturbations T', γ', and ρ' superimposed on the stationary solution obtained above:

$$\frac{\partial T'}{\partial t}-\left[\frac{\beta R T_0}{f^2}\left(k_3+\frac{2\Gamma k_1 k_4}{k_2}\right)+a_1+a_2\right]\frac{\partial T'}{\partial x}-$$
$$-\frac{T_0}{\hat{\rho}_0}\left[U_0 k_1+\frac{\beta R T_0}{f^2}\left(k_3+\frac{\Gamma k_1 k_4}{k_2}\right)-k_3 a_2+a_1\right]\frac{\partial \rho'}{\partial x}-$$
$$-\frac{\Gamma T_0}{\gamma_a k_2}\left[\bar{U}_0-\frac{\Gamma\beta R T_0 k_4}{f^2 k_2}+a_1\right]\frac{\partial\gamma'}{\partial x}=0,$$
$$\frac{\partial\gamma'}{\partial t}+\frac{k_1}{k_2}\left[\bar{U}_0-\frac{\Gamma\beta R T_0 k_4}{f^2}-\frac{k_3 a_1}{k_1}\right]\frac{\partial\gamma'}{\partial x}+\left[\frac{\Gamma\beta g k_1 k_4}{f^2 k_2}+\frac{\gamma_a k_1 a_2}{\Gamma\bar{T}_0}\right]\frac{\partial T'}{\partial x}-$$
$$-\left[\frac{\Gamma\beta g\bar{T}_0 k_1 k_3 k_4}{f^2\hat{\rho}_0 k_2}-\frac{\gamma_a k_1^2 a_2}{\Gamma\hat{\rho}_0}\right]\frac{\partial\rho'}{\partial x}=0,$$
$$\frac{\partial\rho'}{\partial t}-\frac{\beta R\hat{\rho}_0}{f^2}\frac{\partial T'}{\partial x}-\frac{\beta R T_0}{f^2}\frac{\partial\rho'}{\partial x}=0. \qquad (5)$$

Here $a_1=(RT_0k_1/f\hat{\rho}_0)\partial\hat{\rho}_0/\partial y$, $a_2=(\Gamma R T_0/f\gamma_a k_2)\partial\gamma_0/\partial y$, and $\bar{U}_0$ is the average zonal wind calculated from the formula $\bar{U}_0=-(R/f\hat{\rho}_0)\partial\hat{\rho}_0\bar{T}_0/\partial y$. We seek the solution of (5) in the form $(T', \gamma', \rho')=(\tilde{T}, \tilde{\gamma}, \tilde{\rho})\exp[ik(x-ct)]$. From the compatibility condition of the system we obtain the characteristic equation

$$c^3+c^2\left\{\frac{k_1}{k_2}\left[\frac{\beta R T_0}{f^2}(k_2+3\Gamma k_4)-\bar{U}_0\right]+\frac{(1+3k_3)a_1}{k_2}+a_2\right\}+$$
$$+c\left\{\frac{\beta R\bar{T}_0 k_1}{f^2 k_2}\left[\frac{\Gamma\beta R T_0 k_4}{f^2}\left(2+k_3+\frac{\Gamma k_4}{k_2}\left(2k_1-\frac{1}{k_2}\right)\right)-\bar{U}_0(2+\Gamma k_4+3k_3)\right]+\right.$$
$$+\frac{\beta R\bar{T}_0 k_1 a_2}{f^2}-\frac{k_1\bar{U}_0 a_1}{k_2}-\frac{2\bar{U}_0 k_1 a_2}{k_2}-\frac{a_1 a_2}{k_2}+\frac{k_3 a_1^2}{k_2}+$$
$$\left.+\frac{2\Gamma\beta R T_0 k_1^2 k_4 a_2}{f^2 k_2^2}+\frac{\beta R\bar{T}_0 a_1}{f^2 k_2}\left(k_1k_3+2\Gamma k_1 k_4+\frac{2\Gamma k_1 k_3 k_4}{k_2}\right)\right\}+$$
$$+\frac{\beta R\bar{T}_0 k_1^2\bar{U}_0^2}{f^2 k_2}-\frac{\Gamma\bar{U}_0(\beta R\bar{T}_0)^2 k_1^2 k_4}{f^4 k_2}+\frac{2\Gamma^2(\beta R\bar{T}_0)^3 k_1^2 k_3 k_4^2}{f^6 k_2^3}-$$
$$-\frac{\beta R\bar{T}_0 k_1}{f^2 k_2}\left[(3+2k_3)\bar{U}_0 a_2+2a_1a_2+\bar{U}_0 k_3 a_1\right]+$$
$$+\frac{(\beta R\bar{T}_0)^2}{f^4 k_2}\left[\frac{k_1 k_4 a_1\Gamma}{k_2}+\frac{\Gamma k_1 k_4 a_2}{k_2}(3+2k_3+2k_3^2)\right]=0. \qquad (6)$$

The zonal flow is unstable or stable with respect to long waves depending on whether the discriminant of the cubic equation (6) is positive or negative. Figure 1 shows the regions of stability and instability (shaded) of the zonal flow as they depend on the average velocity of that flow and the numerical value of the lapse rate.* We see that the neutral curve passes through the region in which the actual values of γ_0 and $\bar{U}_0$ are concentrated (open circle in Fig. 1), and thus that our model admits of growth of ultralong waves due to breakup of an existing zonal flow. We note (see

*The system has two more stationary solutions, which we shall not consider because they are obviously without physical content.

*Here and in all of the calculations presented below, a_1 and a_2 have been assumed to equal zero. This assumption does not qualitatively affect the nature of the results obtained and has very little effect on quantitative estimates, since the terms that contain a_1 and a_2 or combinations thereof as multipliers are at least an order of magnitude smaller than the remaining terms of Eq. (6).

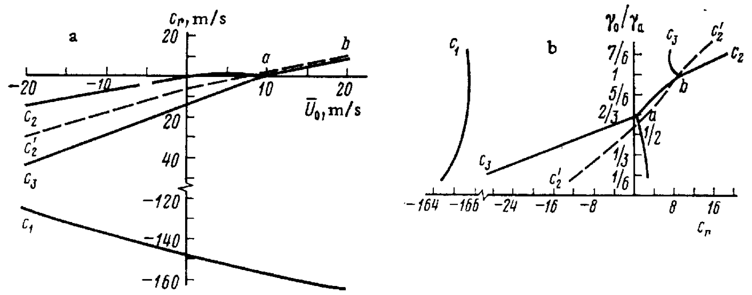

Fig. 2. Real part of phase velocity of ultralong waves c_r as functions of a) average zonal flow velocity $\bar{U}_0$ ($\gamma_0/\gamma_a = 2/3$) and b) temperature stratification parameter γ_0/γ_a ($U_0 = 10$ m/s).

Fig. 1) the existence of a region of absolute stability at -32 m/s $< \bar{U}_0 < 0$, in agreement with the result of [5]. Figure 2 shows how the real part of the phase velocity of the ultralong wave depends on the average zonal-flow velocity $\bar{U}_0$ and the ratio γ_0/γ_a for all three modes described by our model, which corresponds to the three characteristic values (c_1, c_2, c_3). Within the segments ab, the characteristic values c_2 and c_3 become complex-conjugate. The real parts of c_2 and c_3 are positive for real γ_0 and $\bar{U}_0$, i.e., the modes corresponding to them shift toward the east. A growing mode corresponds to the characteristic value c_3 and has a characteristic growth time of five - eight days (depending on the average velocity of the zonal flow) at a wavelength of ~15000 km. The figure shows that the mode corresponding to c_1 is neutral everywhere for the observed values, and that its characteristics are nearly independent of stratification.

The above results indicate that a noncontradictory two-dimensional baroclinic model that describes a broader class of atmospheric motions than the corresponding model in [1, 2] can be derived by averaging the equations of a polytropic atmospheric model over height. We note that in setting $\gamma(x, y, t) = \text{const} \neq \gamma_a$ we were forced to abandon satisfaction of the exact boundary condition (of the heat-input equation) at the surface of the earth, as was done in [4] in derivation of an atmospheric model with two parameters on the vertical. Otherwise we would have an artificial diagnostic relation between the thermohydrodynamic variables that would impose severe limitations on the class of possible atmospheric motions. In the case of our model, the condition $\gamma = \text{const} \neq \gamma_a$ results in merging of the two modes corresponding to the characteristic values c_2 and c_3 into a single mode (c_2', which is represented by the dashed curve in Fig. 2) and, accordingly, in elimination of the instability region shown on the right in Fig. 1. At the same time, we see from Fig. 2 that the phase velocity c_2' does not differ strongly from the actual velocities c_2 and c_3 for realistic values of the lapse rate and the average zonal wind velocity. If, therefore, we consider a model in which there is a stronger instability of nature other than that used to the variability of the parameter γ and if the region of the instability overlaps the region indicated on the right in Fig. 1 to a significant extent, it will be possible to use the simplification $\gamma = \text{const}$.

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