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Dynamics of Ultralong Wave in a Two-Dimensional Baroclinic Atmospheric Model

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An investigation of ultralong waves is made in the planetary-scale geostrophic motion approximation (Kibel number $\sim 10^{-2}$) using the height-averaged equations of hydrothermodynamics, which take into account the horizontal baroclinicity effect. Two types of wave solutions are obtained for the linear problem: (1) fast waves, propagating westward, (2) slow waves, moving eastward. The relationship between the amplitudes of these waves is investigated as a function of the form of the initial perturbation. The stationary regime, which arises in the system due to the action of heat sources and sinks that are both zonal and varying with longitude (because of nonuniform heating of the continents and oceans), is examined. It is shown that the accomodation of the ultralong waves to the long-term climatic variations of external conditions is achieved primarily because of the eastward slow wave component.

As first noted in [1], we must distinguish between two types of large-scale quasigeostrophic motions. According to the terminology proposed in [2], geostrophic motions of the first type include those whose characteristic horizontal scale $L \sim 10^6$ m, while geostrophic motions of the second type are those for which $L \sim 10^7$ m. Besides characteristic horizontal scales, these two types of motions have several other distinguishing features. In particular, the relationship between the vertical component of the relative eddy ξ and the horizontal divergence D for the first type has the form $\zeta \gg D$, while the second $\zeta \approx D$. Accordingly, the filtered equations that are employed to describe these motions are different. Namely, for the description of motions of the second type in the Euler equations we can ignore relative accelerations compared with Coriolis accelerations, i.e., we can replace these equations by the diagnostic relations [1-3]. In this case the equations of continuity and thermodynamics remain unchanged [1-3]. Such equations were used in [4] to study the dynamics of planetary atmospheric perturbations within the framework of the two- and three-level model, taking only vertical baroclinicity into account. However, it already follows from [1] that the expansion of the eddy equation in terms of the Kibel parameter it is necessary to take account of terms, which describe the horizontal baroclinicity, in the case of planetary-scale motions even in the zero approximation. Let us note that the need to take account of terms describing the horizontal baroclinicity effect in studies of planetary atmospheric motions was first pointed out in [5].

The present paper is devoted to an investigation of the dynamics of ultralong waves. The height-averaged geostrophic equations of the second type, which take the horizontal baroclinicity effect into account, are used as the model. The investigation is carried out in the β-plane approximation.

1. Using a procedure for height-averaging the equations of hydrothermodynamics that is analogous to that in [6], we obtain the system of equations

$$\frac{d\mathbf{V}}{dt} - \mathbf{V}\times\mathbf{n}f = -\frac{R}{m}\nabla(Tm) + \mathbf{F}_d,$$

$$\frac{dm}{dt} + m\nabla\cdot\mathbf{V} = 0, \qquad (1)$$

$$\frac{dT}{dT} + \gamma T\Delta\cdot\mathbf{V} = Q + \mu_T\Delta T.$$

Here $\mathbf{V}$ is the horizontal velocity, weight-averaged over a vertical column of air; T is the weight-averaged temperature; $m = p(0, x, y, t)/p_0$; $p_0 = 1000$ mbar; $p(0, x, y, t)$ is the surface pressure; R is the gas constant; c_p is the specific heat of air at constant pressure; $\gamma = R/c_p$; $f = f_0 + \beta y$ is the Coriolis parameter; $\mathbf{n}$ is a unit vector normal to the earth's surface; $\mu_T = 3\cdot 10^6$ m^2/sec is the horizontal turbulent heat diffusion coefficient; $\mathbf{F}_d$ is the frictional force; Q are nonadiabatic heat sources. (Let us note that the third equation of the system (1) was first obtained in [7]).

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The geostrophic equations of the second type, corresponding to the system (1), are

$$-\mathbf{V}\times\mathbf{n}f=-\frac{R}{m}\nabla(Tm)+\mathbf{F}_d,$$
$$\frac{dm}{dt}+m\nabla\cdot\mathbf{V}=0, \qquad (2)$$
$$\frac{dT}{dt}+\gamma T\nabla\cdot\mathbf{V}=Q+\mu_T\Delta T.$$

Let us show that (for an adiabatic nonviscous atmosphere) the solutions of the linearized system (1) at sufficiently small wave numbers differ very little from the corresponding solutions of the linearized system (2).

We linearize the system (1) with respect to $T_0=T_0(y)$, $m_0=1$, $U_0=-(R/f_0)\partial T_0/\partial y$. We look for the solutions for perturbations in the form

$$\begin{pmatrix} u' \\ v' \\ m' \\ T' \end{pmatrix}=\begin{pmatrix} \hat{u} \\ \hat{v} \\ \hat{m} \\ \hat{T} \end{pmatrix} e^{i(kx-\omega t)}. \qquad (3)$$

The fact that only motions along the x axis are considered does not alter the basic conclusion, but it does simplify the calculations considerably. After substituting (3) into the equations of the linear system obtained from (1) and using the equation for the velocity eddy, we will have

$$i\varepsilon(U_0k-\omega)\hat{u}-f_0\hat{u}+ikRT_0\hat{m}+ikR\hat{T}=0,$$
$$ikf_0\hat{u}-\varepsilon(U_0k-\omega)k\hat{v}+\beta\hat{v}-iR\frac{\Delta T_0}{\partial y}k\hat{m}=0, \qquad (4)$$
$$ik\hat{u}+i(U_0k-\omega)\hat{m}=0,$$
$$ik\gamma T_0\hat{u}+\frac{\partial T_0}{\partial y}\hat{v}+i(U_0k-\omega)\hat{T}=0.$$

The parameter ε assumes values of 1 or 0. The value $\varepsilon=0$ means that the first equation of the system (1) is replaced by the diagnostic relation

$$\mathbf{V}\times\mathbf{n}f=\frac{R}{m}\nabla(Tm). \qquad (5)$$

The system (4) has a nontrivial solution if its determinant is equal to zero. Expanding the determinant and introducing the notations

$$kU_0-\omega=Z,\quad (1+\gamma)RT_0=c_0^2,\quad -R\frac{\partial T_0}{\partial y}=U_0f_0, \qquad (6)$$

we obtain

$$(\varepsilon Z^2-k^2c_0^2)\left(\varepsilon Z-\frac{\beta}{k}\right)Z-f_0^2(Z-U_0k)^2=0. \qquad (7)$$

For $\varepsilon=0$ Eq. (7) reduces to a second order equation

$$Z^2-\frac{c_0^2\beta k+2U_0f_0^2k}{f_0^2}Z+U_0^2k^2=0. \qquad (8)$$

Its solution is

$$Z_{1,2}=\frac{c_0^2\beta k}{2f_0^2}\pm\sqrt{\frac{c_0^4\beta^2k^2}{4f_0^4}+\frac{c_0^2\beta U_0k^2}{f_0^2}}+U_0k \qquad (9)$$

or for ω

$$\omega_{1,2}=-\frac{c_0^2\beta}{2f_0^2}\left(1\pm\sqrt{1+\frac{4f_0^2U_0}{c_0^2\beta}}\right)k. \qquad (10)$$

Let us find the approximate values of the roots of Eq. (7) for $\varepsilon=1$. If $|Z|\gg\beta/k$, $|Z|\gg U_0k$, then we can write Eq. (7) thus:

$$Z^2(Z^2-k^2c_0^2)-f_0^2Z^2=0 \qquad (11)$$

and

$$\omega_{1,2}=\left(U_0\pm\sqrt{c_0^2+\frac{f_0^2}{k^2}}\right)k; \qquad (12)$$

if $U_0k\ll|Z|\ll kc_0$, then

$$\left(Z-\frac{\beta}{k}\right)k^2c_0^2+f_0^2Z=0 \qquad (13)$$

and

$$\omega_3=U_0k-\frac{\beta k}{k^2+f_0^2/c_0^2}; \qquad (14)$$

if $|Z|\ll U_0k$, $|Z|\ll\beta/k$, $|Z|\ll c_0k$, then

$$Z-\frac{f^2U_0^2k}{c_0^2\beta}=0 \qquad (15)$$

and

$$\omega_4=\left(U_0-\frac{f_0^2U_0^2}{c_0^2\beta}\right)k. \qquad (16)$$

Let us note that the slow waves ω_3(10) and ω_i(16), moving with a velocity that is much smaller than the velocity of the Rossby waves, correspond to the baroclinic mode of the two-level atmospheric model [8].

Figure 1 shows the dispersion curves for the cases $\varepsilon=0$ and $\varepsilon=1$. For $k<1.5\cdot10^{-7}$ m^{-1} the shape of curves 1 and 2 in Fig. 1, a almost coincides with the shape of curves 3 and 4 in Fig. 1, b; this can also be seen from Eqs. (10), (14), and (16). Thus, we have shown that in the case of the longest waves the system (2) is a good approximation of the system (1).

2. Let us consider a more general model that describes the behavior of ultralong waves, i.e., we discard the assumption $m_t=1$. Let us first examine the case of a nonviscous adiabatic atmosphere. Eliminating the velocity from the system

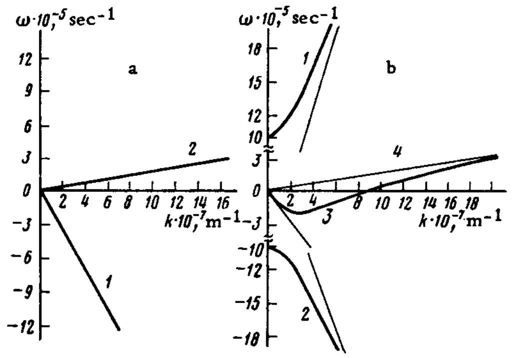

Fig. 1. Dispersion curves from Eq. (7): a - case of $\varepsilon = 0$, 1 - "barotropic mode", representing the limiting Rossby wave, 2 - "baroclinic mode"; b - $\varepsilon = 1$, 1, 2 - acoustic waves, 3 - Rossby wave, 4 - "baroclinic mode".

(2), we will have

$$\frac{\partial m}{\partial t} - \frac{\beta R}{f_0^2}\frac{\partial Tm}{\partial x} = 0,$$
$$\frac{\partial T}{\partial t} + \frac{(1+\gamma)}{f_0 m^2} RT(m, Tm) - \frac{\gamma\beta RT}{f_0^2 m}\frac{\partial Tm}{\partial x} = 0. \qquad (17)$$

here $(m, Tm) = \partial m/\partial x \partial Tm/\partial y - \partial m/\partial y \partial Tm/\partial x$.

The system (17) has stationary solutions m_0, T_0, which can be obtained by equating $\partial m/\partial t$ and $\partial T/\partial t$ to zero:

$$\frac{\partial T_0 m_0}{\partial x} = 0, \quad \frac{\partial m_0}{\partial x}\frac{\partial T_0 m_0}{\partial y} = 0. \qquad (18)$$

From this we find two types of stationary solutions.

1) solution $T_0(x, y) = \dfrac{\text{const}}{m_0(x, y)}, \quad U_0 = 0, \quad v_0 = 0,$ (19)

2) solution $m_0 = m_0(y), T_0 = T_0(y), U_0 = -\dfrac{R}{f_0 m_0}\dfrac{\partial T_0 m_0}{\partial y}, v_0 = 0.$ (20)

The first stationary solution has no physical meaning, and we will ignore it. With respect to the second solution, taking into account that $m_0 = 1 + O(10^{-2})$, the formula for U_0 can be written thus: $U_0 = -(R/f_0)\partial T_0/\partial y$. This last relation is nothing more than the thermal wind equation, integrated with respect to height. It follows from this, by the way, that in this model it is impossible to eliminate the quantity U_0, by changing to a moving reference system since U_0 is not actually the velocity itself, but its shearing between ground level and the height of the homogeneous atmosphere.

Let us linearize the system (17) with respect to the stationary solution (20)

$$\frac{\partial m'}{\partial t} - \frac{\beta RT_0}{f_0^2}\frac{\partial m'}{\partial x} - \frac{\beta Rm_0}{f_0^2}\frac{\partial T'}{\partial x} = 0,$$
$$\frac{\partial T'}{\partial t} - \frac{T_0}{m_0}\left[(1+\gamma)U_0 + \frac{(1+\gamma)RT_0}{f_0 m_0}\frac{\partial m_0}{\partial y} - \frac{\gamma\beta RT_0}{f_0^2}\right]\frac{\partial m'}{\partial x} - \left[\frac{(1+\gamma)RT_0}{f_0 m_0}\frac{\partial m_0}{\partial y} + \frac{\gamma\beta RT_0}{f_0^2}\right]\frac{\partial T'}{\partial x} = 0. \qquad (21)$$

We look for the solution of (21) in the form

form

$$\begin{pmatrix} m' \\ T' \end{pmatrix} = \begin{pmatrix} \hat{m}(y) \\ \hat{T}(y) \end{pmatrix} e^{ik(x-ct)}. \qquad (22)$$

Substituting (22) into (21), we obtain

$$[c + b_1 T_0]\hat{m} + b_1 \hat{T} = 0,$$
$$[(1+\gamma)T_0 U_0 + (b_2 + b_3)T_0]\hat{m} + [c + b_2 + b_3]\hat{T} = 0 \qquad (23)$$

where

$$b_1 = \frac{\beta R}{f_0^2} \approx 0.46\ \text{m}\cdot\text{sec}^{-1}\cdot\text{deg}^{-1}\quad b_2 = \frac{(1+\gamma)RT_0}{f_0 m_0}\frac{\partial m_0}{\partial y} \sim 5.8\ \text{m}\cdot\text{sec}^{-1}$$
$$b_3 = \frac{\gamma\beta RT_0}{f_0^2} \approx 34.4\ \text{m}\cdot\text{sec}^{-1}.$$

The system (23) has a nontrivial solution if its determinant is equal to zero. From this condition we find

$$c = -\frac{b_1 T_0 + b_2 + b_3}{2} \mp \left[\frac{(b_1 T_0 + b_2 + b_3)^2}{4} + (1+\gamma) b_1 T_0 U_0\right]^{1/2}. \qquad (24)$$

Let us note that the velocity value according to (24) differs very little from its value using Eq. (10), i.e., taking account of the variation of the surface pressure with latitude only slightly affects the propagation velocity of ultralong waves in the earth's atmosphere. Since $U_0(y) > 0$, it follows from (24) that planetary motions are always neutral. The phase velocities of the waves do not depend on the wave number k and are approximately equal to: $c_1 \approx -150$ m/sec, $\tau_1 \approx 2$ days, $c_2 \approx 9$ m/sec, $\tau_2 \approx 31$ days (for $U_0 \approx 10$ m/sec); $c_1 \approx -175$ m/sec, $\tau_1 \approx 1.8$ days, $c_2 \approx 17$ m/sec, $\tau_2 \approx 15$ days (for $U_0 \approx 20$ m/sec) (τ is the time for the wave to travel around the earth). The velocity c_2 of the slow component agrees in order to magnitude and sign with the velocity value obtained in [4]. If it is assumed that the average wind velocity in the atmosphere is $U_0 \approx 20$ m/sec, then the result $\tau_2 \approx 15$ days for the temperature wave matches quite well the observational data given in [9].

3. Let us examine the relationship between the amplitudes of the waves, moving with velocities c_1 and c_2, as a function of the form of the initial perturbation. The solution of the system (21) can

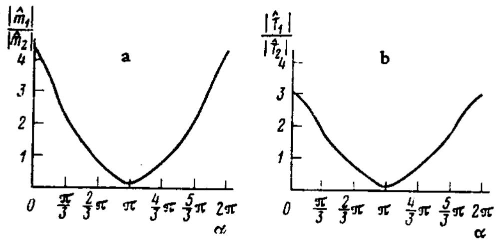

Fig. 2. Dependence of the ratio of the modulus of the amplitude of the fast wave component to the modulus of the amplitude of the slow wave component on the value of the angle α between $m'(x, y, 0)$ and $T'(x, y, 0)$: *a* – for $|\hat{m}_1|/|\hat{m}_2|$, b – for $|\hat{T}_1|/|\hat{T}_2|$.

be written as follows:

$$m' = \hat{m}_1 e^{ik(x-c_1t)} + \hat{m}_2 e^{ik(x-c_2t)},$$
$$T' = \hat{T}_1 e^{ik(x-c_1t)} + \hat{T}_2 e^{ik(x-c_2t)}. \qquad (25)$$

We take the initial conditions in the form

$$m'(x,y,0) = A_0(y) e^{ikx}, \quad T'(x,y,0) = B_0(y) e^{i(kx+\alpha)}. \qquad (26)$$

Substituting (25) into (26) and (21), we obtain a system of four linear algebraic equations with four unknowns, through the solution of which we can obtain the following expressions for the ratios of the moduli of the amplitudes of waves 1 and 2:

$$\frac{|\hat{m}_1|}{|\hat{m}_2|} = \left[\frac{(g_1B_0\cos\alpha + (b_1T_0+c_2)A_0)^2 + b_1^2B_0^2\sin^2\alpha}{(b_1B_0\cos\alpha + (b_1T_0+c_1)A_0)^2 + b_1^2B_0^2\sin^2\alpha}\right]^{1/2},$$

$$\frac{|\hat{T}_1|}{|\hat{T}_2|} = \left[\frac{(F_0A_0 + (b_2+b_3+c_2)B_0\cos\alpha)^2 + (b_2+b_3+c_2)^2B_0^2\sin^2\alpha}{(F_0A_0 + (b_2+b_3+c_1)B_0\cos\alpha)^2 + (b_2+b_3+c_1)^2B_0^2\sin^2\alpha}\right]^{1/2}, \qquad (27)$$

where $F_0 = (b_2+b_3+(1+\gamma)U_0)T_0$.

The dependence of the ratios $|\hat{m}_1|/|\hat{m}_2|$ and $|\hat{T}_1|/|\hat{T}_2|$ on α for the values $A_0 \approx 0{,}02$, $B_0 = 5$ K, $U_0 = 20$ m/sec is shown in Fig. 2.

4. Let us introduce heat sources into the problem. The system of equations is written

$$\frac{\partial m}{\partial t} - \frac{\beta R}{f_0^2 m}\frac{\partial T_m}{\partial x} = 0,$$
$$\frac{\partial T}{\partial t} - \frac{(1+\gamma)RT}{f_0 m}(m, Tm) - \frac{\beta R\gamma T}{f_0^2 m}\frac{\partial Tm}{\partial x} = \mu_T\Delta T + Q. \qquad (28)$$

The function Q consists of two parts: $Q = Q_1 + Q_2$, and $|Q_2|/|Q_1| \ll 1$; Q_1 is determined from Newton's formula:

$$Q_1 = H(T^* - T), \qquad (29)$$

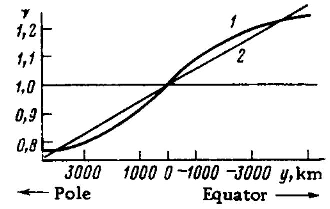

Fig. 3. The empirical function $q(y)$.

where $T^* = \bar{T}(y)q(y)$ is the temperature in the radiation equilibrium state; $\bar{T}(y)$ is the average temperature; $H = 8.3\cdot10^{-7}$ sec^{-1}; $q(y)$ is an empirical function (see [10]). The graph of $q(y)$, provided in the cited paper, is reproduced here (Fig. 3, curve 1). Considering the midlatitude region, to simplify the calculations we approximate $q(y)$ by a linear function (Fig. 3, curve 2)

$$q(y) = 1 - ry \qquad (30)$$

(the origin of the coordinate system is chosen in the midlatitudes, $r = 3\cdot10^{-8}$ m^{-1}). Q_2 describes the heat sources and sinks due to the temperature difference of the sea and land. We assume (taking account of the arrangement of the continents and oceans) that

$$Q_2 = S(T^{**} - T), \qquad (31)$$

where $T^{**} = \bar{T}(y)q_1(x,y)$, $q_1(x,y) = 1 + \varepsilon(y)\sin k(x-\varphi)$, $k = 2\pi n/L$, $n = 0, 1, 2, \ldots$, L is the length of a circle of latitude; $S \sim 10^{-7}$ sec^{-1}; $\varepsilon \sim 0.04$.

Let us find the solution of the system (28), looking for it in the form

$$T = \bar{T}(y) + T'(x,y), \quad m = 1 + m'(x,y), \qquad (32)$$

and $|T'|/|\bar{T}| \ll 1$, $m' \ll 1$. Restricting the discussion to quantities of first-order of smallness, we obtain the following system:

$$\bar{T}\frac{\partial m'}{\partial x} + \frac{\partial T'}{\partial x} = 0,$$
$$-\frac{(1+\gamma)R\bar{T}}{f_0}\frac{\partial \bar{T}}{\partial y}\frac{\partial m'}{\partial x} = \mu_T\Delta T' - (H+S)T' + S\bar{T}\varepsilon\sin k(x-\varphi) = 0,$$
$$\mu_T\frac{\partial^2 T}{\partial y^2} + H\bar{T}(q(y)-1) = 0. \qquad (33)$$

We make the assumption that $\left|\frac{\partial^2 T'}{\partial x^2}\right| \Big/ \left|\frac{\partial^2 T'}{\partial y^2}\right| \gg 1$.

In this case the first term on the right side of the second equation of the system (33) reduces to $\mu_T(\partial^2 T'/\partial x^2)$. Now, eliminating $\partial m'/\partial x$ from the second equation of the system (33) with the aid of

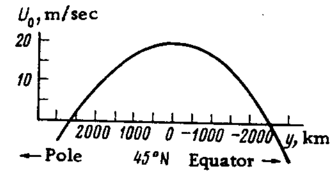

Fig. 4. Dependence of the average zonal wind velocity U_0 on y.

the first equation, we obtain an ordinary differential equation for T' with coefficients that depend on y as on a parameter. The function $T(y)$ is determined from the third equation. The third equation of the system (33) for $q(y)$ taken in the form of (3) is the Airy equation

$$\bar{T}'' - \frac{Hr}{\mu_T} y\bar{T} = 0. \tag{34}$$

For convenience we introduce the new coordinate $y^0 z (Kr/\mu_T)^{\frac{1}{3}} y$. Then (34) becomes

$$\bar{T}''(y^0) + y^0 \bar{T}(y^0) = 0. \tag{35}$$

Its solution will be

$$\bar{T} = a_1 w_1 + a_2 w_2. \tag{36}$$

Here w_1 and w_2 are linearly independent solutions of the Airy equation, corresponding to the initial conditions

$$w_1(0) = 1,\ w_1'(0) = 0,$$
$$w_2(0) = 0,\ w_2'(0) = 1.$$

By specifying the initial conditions for $\bar{T}$ we find the constants a_1 and a_2. For winter they are equal to 259 and 31, respectively. In the interval $-1 < y^0 < 1$ $(-5\cdot 10^3$ km $< y < 5\cdot 10^3$ km$)$ the solution (36) can be approximated by the first terms of the expansion of the functions w_1 and w_2 in powers of y

$$\bar{T}(y) = a_1\left[1 + \frac{1}{6}\frac{Hr}{\mu_T} y^3 + \frac{1}{180}\left(\frac{Hr}{\mu_T}\right)^2 y^6\right] - a_2\left[\left(\frac{Hr}{\mu_T}\right)^{1/3} y + \frac{1}{12}\left(\frac{Hr}{\mu_T}\right)^{4/3} y^4\right] \tag{37}$$

Hence, the average zonal wind velocity U_0 is

$$U_0 = -\frac{R}{f_0}\frac{\partial T}{\partial y} = a_2\left(\frac{Hr}{\mu_T}\right)^{1/3}\frac{R}{f_0} + \frac{R}{f_0}\left[a_2\left(\frac{Hr}{\mu_T}\right)^{4/3}\frac{y^3}{3} - \frac{a_1}{2}\frac{Hr}{\mu_T}y^2 - \frac{a_1}{30}\left(\frac{Hr}{\mu_T}\right)^2 y^5\right]. \tag{38}$$

The dependence of U_0 on y is shown in Fig. 4 and it agrees, at least qualitatively, with the wind behavior in the midlatitudes.

Substituting, now, the found function $T(y)$ into the equation for the temperature perturbations, we obtain its solution

$$T' = B_0 \sin k(x - \varphi_1), \tag{39}$$

where

$$B_0 = \frac{S\bar{T}\varepsilon(y)}{\mu_T\left[\frac{k^2(1+\gamma)^2R^2}{\mu_T f_0^2}\left(\frac{\partial\bar{T}}{\partial y}\right)^2 + \left(k\eta + \frac{H+S}{\mu_T}\right)^2\right]^{1/2}}, \tag{40}$$

$$k\varphi_1 = k\varphi - \operatorname{arctg}\frac{\mu_T k^2 + H + S}{\left|\frac{k(1+\gamma)R}{f_0^2}\frac{\partial\bar{T}}{\partial y}\right|} + \frac{1}{2}\pi. \tag{41}$$

The phase shift between $k\varphi_1$ and $k\varphi$ is equal to 60°, which agrees with the result of [11], obtained by another method.

Finally, we find from the first equation of the system (33)

$$m' = -\frac{B_0}{\bar{T}}\sin k(x - \varphi_1). \tag{42}$$

It is seen that the temperature wave and the surface pressure wave are in opposite phase, and the relationship between their amplitudes is

$$\bar{T}A_0 = B_0$$

(A_0 is the amplitude of the surface pressure wave, B_0 is the amplitude of the temperature wave).

5. Since the distribution of heat sources and sinks varies during the year, the stationary solution of the system (28) in the form (39), (42), corresponding to the distribution q_1, will no longer satisfy the system (33) for some new distribution q_1'. This raises the question of how the meteorological fields, accommodating to the new distribution of heat sources and sinks, evolve?

We assume that the function Q_2 at time $t=0$ suddenly changes by a small amount. Let us assume, for example,

$$q_1'(x, y) = (\varepsilon + \delta\varepsilon)\sin k(x - \varphi - \delta\varphi) + 1. \tag{43}$$

Solving the linearized system (28), we find for the new distribution of heat sources and sinks

$$T' = B_{0\delta}e^{ik(x-\varphi_\delta)} + \hat{T}_{01}e^{-\sigma_1 t}e^{ik(x-c_1t-\varphi-\delta\varphi)} + \hat{T}_{02}e^{-\sigma_2 t}e^{ik(x-c_2t-\varphi-\delta\varphi)}, \tag{44}$$

$$m' = -\frac{B_{0\delta}}{\bar{T}} e^{ik(x-\varphi_\delta)} + \hat{m}_{01} e^{-\sigma_1 t} e^{ik(x-c_1 t-\varphi-\delta\varphi)} + \hat{m}_{02} e^{-\sigma_2 t} e^{ik(x-c_2 t-\varphi-\delta\varphi)}, \tag{45}$$

where

$$B_{0\delta} = \frac{S\bar{T}(\varepsilon+\delta\varepsilon)}{\left[(\mu_T k^2+H+S)^2 + \frac{k^2(1+\gamma)^2 R^2}{f_0^2}\left(\frac{\partial\bar{T}}{\partial y}\right)^2\right]^{1/2}}, \tag{46}$$

$$k\varphi_\delta = k(\varphi+\delta\varphi) - \operatorname{arctg}\frac{\mu_T k^2+H+S}{\frac{k(1+\gamma)R}{f_0}\left|\frac{\partial\bar{T}}{\partial y}\right|} + \frac{1}{2}\pi, \tag{47}$$

$$c_{1,2} \approx -\frac{b_1\bar{T}+b_3}{2} \mp \left[\frac{(b_1\bar{T}+b_3)^2}{2} + (1+\gamma)b_1\bar{T}U_0 - \frac{\left(\mu_T k + \frac{H+S}{k}\right)^2}{4}\right]^{1/2}, \tag{48}$$

$$\sigma_{1,2} \approx \frac{1}{2}(\mu_T k^2+H+S) \mp \frac{(b_1\bar{T}-b_3)(\mu_T k^2+H+S)}{4\left[\frac{(b_1\bar{T}+b_3)^2}{4} + (1+\gamma)\bar{T}U_0 b_1\right]^{1/2}}. \tag{49}$$

Substituting the numerical values of the parameters into (49), we find that the characteristic damping time for the fast wave is $\sigma_1^{-1} \sim 30$ days and for the slow wave $\sigma_2^{-1} \sim 12$ days (for $U_0 \approx 20$ m/sec).

The numerical values of the coefficients $\hat{T}_{01}$, $\hat{T}_{02}$, $\hat{m}_{01}$, $\hat{m}_{02}$ are found from the initial conditions. Finally, considering the stationary solution (39), (42) (corresponding to q_1) as the initial conditions for the system (28) with a new distribution of heat sources and sinks, described by q_1', we find that the ratio of the modulus of the amplitude of the fast component to the modulus of the slow component at the initial instant of time is no greater than 10^{-1} (within an accuracy of 10^{-2}). Thus, primarily slow waves will travel in the atmosphere for a change in the distribution of heat sources and sinks by some amount. The initial amplitude of these waves is proportional to the magnitude of this sudden change. In the limit as $t \to \infty$ the temperature and surface pressure distribution approaches a steady-state condition that corresponds to the heat source and sink distribution q_1'. It can be concluded from this that the accommodation of the atmosphere to long-period climatic changes of the external conditions occurs by means of the slow-wave component. Moreover, taking into account that the atmosphere at each given instant of time is close to the state defined by the stationary solution of the problem, we arrive at the conclusion that for perturbations whose characteristic horizontal dimension is comparable to the earth's radius the primary type of wave motions will be the slow wave with velocities of the order of U_0 toward the east.

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