APPLICATION OF THE MODIFIED ERTEL'S POTENTIAL VORTICITY TO INVESTIGATION OF SOUTH HEMISPHERE CIRCULATION

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ABSTRACT

A arbitrariedade que existe em definição geral da vorticidade potencial de Ertel (EPV) é usada para conseguir vorticidade potencial (PV) modificada " optimal ", no sentido de minimizar a rapidez de mudança da PV com tempo devido ao atrito e a aquecimento diabatico. Foi obtido que novo PV modificado e alguns dos seus funcionais são quasi-conservadores em processos climáticos, e podem ser usados com alguma vantagem em investigações de regimes climaticos. Aplicando funcionais tais como a carga do vorticidade, e entropia informativa da distribuição da massa de ar sob vorticidade potencial modificado de Ertel ao estudo de comportamento temporal das estatisticas da circulação para os meses de janeiro e de julho detectamos um crescimento progressivo da diferença da temperatura do ar entre equador e pólo na superfície do Hemisfério Sul durante os 1980s.

INTRODUCTION

The well-known freedom available in Ertel's (1942) potential vorticity definition allows one to arrive at a pair of (q, $\chi(\Theta)$)-variables [q is the modified potential vorticity (MPV), $\chi(\Theta)$ the appropriately taken function of potential temperature Θ which enters MPV-definition], such that the air mass enclosed within solenoids formed by intersection between the families of equiscalar surfaces q=const and χ=const is nearly preserved despite the influence of diabatic heating and turbulent friction. It is hypothesised and prove that in (q,χ)-coordinates the long-term atmospheric climate processes admit a simple statistical description. It is shown in this paper, that for an atmosphere the reference state exists. This reference state is described by a probability density function μ. The function μ depends on MPV only and describes a "climate background noise" corresponding to the regime of complete statistical equilibrium of the atmosphere. The real atmosphere is close to the reference state (Kurgansky and Pisnichenko, 2000).

METHOD AND DATA

Principal equations.

Following Obukhov (1964) we take $\chi(\Theta)=-p^*(\Theta)/g$, where p* is a reference pressure dependence upon Θ, and g is the gravity acceleration, and apply the term "potential vorticity" to the quantity

$$q = \rho^{-1}\mathbf{Z}\cdot\nabla\left(-p^*(\Theta)/g\right), \qquad (1)$$

having an order of magnitude of the Coriolis parameter (10^{-4} s^{-1}). Here, ρ is the density and Z is the absolute vorticity. For the p* we use the function

$$p^* = A - B\cdot arctg\left[C\left(\Theta-\Theta_0\right)\right], \qquad (2)$$

which corresponds to Cauchy distribution in statistics (see e.g. Kurgunsky and Tatarskaya,1987; Pisnichenko and Kusgansky,1996). Here, A, B, C and Θ_0 are some constants.
From (1) it follows that the integral

$$Z_A = \iiint_V q\rho \quad dV \equiv \iiint_V [\mathbf{Z}\cdot\nabla(-p^*(\Theta)/g)]dV\,,$$

taken over the volume V of the atmosphere becomes the well-defined finite quantity and is named the "atmospheric vortex charge", which stresses an analogy with the electric charge conservation law in electrodynamics. When V contains the hemispheric portion of the atmosphere one arrives at the atmospheric vortex charge either over the Northern Hemisphere Z_{NH} or over the Southern Hemisphere Z_{SH}, such that $Z_A= Z_{NH}+ Z_{SH}$.

Dynamical fundamentals

Under the influence of diabatic heating $\dot{\Theta}$ and frictional forces $\mathbf{F}$ the potential vorticity q transforms according to the equation (e.g. Haynes and McIntyre 1990)

$$\dot{q} = \rho^{-1}\left[\mathbf{Z}\cdot\nabla\dot{\chi} + \nabla\chi\cdot\nabla\times\mathbf{F}\right], \qquad (3)$$

where a dot above variables denotes the material time-derivative. Equation (3) keeps it form under the transformation: $\chi\Rightarrow\chi^*=\Phi(\chi)$, $q\Rightarrow q^*=\Phi' q$. Isoscalar surfaces q=const and χ=const divide the atmosphere into (q, χ) solenoids, along which air masses flow during adiabatic and frictionless processes. A solenoidal vector $A=\nabla q\times\nabla\chi$ obeys the equation

$$\frac{D}{Dt}\frac{\mathbf{A}}{\rho} - \left(\frac{\mathbf{A}}{\rho}\cdot\nabla\right)\mathbf{v} = \frac{1}{\rho}\nabla\times\left(\dot{q}\nabla\chi - \dot{\chi}\nabla q\right),$$

that readily follows from Ertel's commutative formula (e.g. Hollmann 1964). The vector A is frozen in a fluid not only when $\dot{q} = \dot{\chi} = 0$, but under less restrictive assumption of

$$\dot{q}\nabla\chi - \dot{\chi}\nabla q = -\nabla B, \qquad (4)$$

where B=B is a scalar function of position vector x and time t.

In order (4) to be valid for another pair of variables (q^{**}, χ^{**}) with $B^{**}=B^{**}(q^{**},\chi^{**},t)$, the canonical transformation (see, Leech, 1958) $q^{**}=f(\chi)q$, $\chi^{**}=h(\chi)$, $f=n^{-2}h'$ has to be performed, with the Jacobian determinant $\partial(q^{**},\chi^{**})/\partial(q,\chi)=n^{-2}$. Here f and h are arbitrary differentiable functions and n is a numerical constant. This is consistent with the above-written demand for preservation of a covariant form of (3)demands for preservation of the covariant form of (3) in the case of $[\Phi']^2=[h']^2 =n^2$ that is when $\chi^{**}=\chi^*= \pm\chi/n$. Formally, based on Lorenz' (1955) ideas, one may suppose the existence of a weightless perfect gas underground. Here, all isentropic surfaces could be regarded as closed surfaces, with PT values covering a range $\Theta_{min}\leq\Theta<\infty$, where Θ_{min} is the minimum PT value in the atmosphere. The entire atmospheric mass lies well above the isentropic surface Θ_{min}=const. So, by χ-definition as a pressure-like variable, it is natural to assume that $\chi^*(\Theta_{min})=\chi(\Theta_{min})$. Consequently $n=\pm1$, which shows that $\chi^{**}\equiv\chi^*\equiv\chi$. We conclude, that (3,4) could be fulfilled simultaneously, if at all, for a single choice of χ. This choice corresponds to the case of an "optimal" potential vorticity modification.

Basic statistical arguments

Consider the quantity $\mu(q,\chi)\, dqd\chi$ equal the part of the total atmospheric mass, enclosed in the solenoid formed by intersection of the surfaces q, q+dq=const and χ,χ+dχ=const. If $\iint \mu(q,\chi)\, dqd\chi=1$, where an integration is taken over all q and χ values, then $\mu(q,\chi)$ function may be regarded as the probability density of q and χ values for a randomly chosen air particle (Obukhov 1964). The atmospheric vortex charge equals $Z_A=m_AQ_A$, where m_A is the total atmospheric mass, and $Q_A=\int q\mu(q,\chi)dqd\chi$ is the first momentum of $\mu(q,\chi)$ distribution. The $\mu(q,\chi)$ distributions were calculated separately for the Northern Hemisphere (NH) and the Southern Hemisphere (SH). They are $\mu_{NH}(q,\chi)$ and $\mu_{SH}(q,\chi)$, respectively

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Reference distribution

We seek a stationary reference air mass distribution μ between infinitely thin, quasi-zonally oriented (q,χ)-solenoids, when the condition (4) for theirs mass preservation is satisfied and a corresponding invariant measure in (q,χ)-functional space is specified. Following classical arguments by Gibbs (1902) and taking into account the principle of "potential vorticity substance" conservation, the most simple and fundamental choice is to set

$\mu=\mu_B(q, \chi)=\mu_0 \exp\{-q/Q\}$.

Integration over all q values gives $\int \mu_0 Q\, d\chi =1$, and, as a consequence

$\mu_B(q)=\int\mu_B(q,\chi)d\chi =\exp\{-q/Q\} \int\mu_0 d\chi =Q^{-1} \exp\{-q/Q\}$.

Further on, $\mu(q, \chi)$ is reduced to the 1D probability density $\mu(q)=\int\mu(q,\chi)\, d\chi$ too.

The following statistical approach to the problem of searching for the optimal PV modification, is suggested. Let us introduce the informational entropy defined according to Shannon and von Neumann (e.g. Katz 1967) for description of air mass statistical distribution on PV and χ values. It can be interpreted as a general measure of degree of uncertainties in PV and χ values for a randomly chosen air parcel. Reference distribution supplies the conditional maximum of informational entropy provided the total amount of modified potential vorticity substance, i.e. ‘atmospheric vortex charge’, is kept constant. The informational entropy deficit, taken as a difference between the maximum possible and actual information entropy values, tends to vanish when canonical (PV, χ) coordinates are used.

For both hemispheres separately the distribution

$\mu_B(q)=|Q|^{-1}\exp(-q/Q)$, (6)

supplies the maximum of the informational entropy $H=-\int\mu\ln\mu dq$, provided

$Q=\int q\mu_B(q)dq=\int q\mu(q)dq$, (7)

holds. The informational entropy maximum value is equal to

$H_B=-\int\mu_B\ln\mu_B dq=\ln|Q|+\text{const}$, (8)

and the informational entropy deficit

$\Delta H=H_B-H$ (9)

characterises the degree of closeness of $\mu(q)$ and $\mu_B(q)$ distributions.

“Optimal” potential vorticity modification

When seeking the “canonical” function p_0^* we gave variations in C and Θ_0 in (2). It was adopted that A=681 pa, B=(2/π)A=433.5 hPa. ECMWF data for 1980-89 were used and for every year the January and July monthly-mean distributions $\mu(q)$ have been calculated for 64 different pairs of C and Θ_0 values for NH and SH, separately. The values of Q, H, H_B and $\Delta H=H_B-H$ have been calculated and then averaged over the entire 10-years period. As it is seen in Fig. 1, $\Delta H=\Delta H(C,\Theta_0)$ attains minimum values in the vicinity of a line given by the linear regression equation $\Theta_0-292.55=321.4\times(C-0.04614)$. Here, Θ_0 is expressed in Kelvin degrees (K) and C is given in K^{-1}. Figure 2 shows that the position of minimum for the corresponding standard deviations of ΔH coincides with that of the mean ΔH values. This confirms that that we arrive at a statistically stable state of minimum. For a further analysis the concrete values of Θ_0=293 K and C=0.04614 K^{-1}, lying close to the regression line, have been chosen. Figure 3 demonstrates evident closeness between $\mu(q)$ and $\mu_B(q)$ distributions for such an optimal PV modification. Here, monthly-mean statistics for January 1980, NH have been used. Some systematic deviations occur only at high PV values.

RESULTS AND DISCUTION

Informational entropy interannual variationas

Temporal behaviour of the informational entropy H for Θ_0=293 K and C=0.04614 K^{-1} within the 1980-89 period is presented in Fig. 4. In the following, the notations $Y_1=H_{JAN}^{NH}$, $Y_2=H_{JAN}^{SH}$, $Y_3=H_{JUL}^{NH}$, $Y_4=H_{JUL}^{SH}$ are introduced. Correlation matrix $r_{ij}=r(Y_i,Y_j)$, i,j=1,2,3,4 calculated on the basis of these data is given by

$$\mathbf{r}=\begin{pmatrix} 1 & 0.09 & 0.03 & 0\,26 \\ 0.09 & 1 & -0.11 & 0.82^* \\ 0.03 & -0.11 & 1 & -0.11 \\ 0.26 & 0.82^* & -0.11 & 1 \end{pmatrix},$$

where the only values marked by an asterisk are significant. Figure 4 demonstrates similar behaviour of Y_1 and Y_2 curves but the corresponding correlation coefficient is negligible. This originates from essential linear trends incorporated in informational entropy values. The corresponding linear regression equations $Y_i=a_iX+b_i$, i=1,2,3,4, where X denotes years and varies in the range of 80-89, are given in Table 1.

TABLE 1

i	a_i	b_i
1 (NH, January)	-0.000685	0.725
2 (SH, January)	0.00176	0.501

3 (NH, July)	0.0000242	0.609
4 (SH, July)	0.00147	0.544

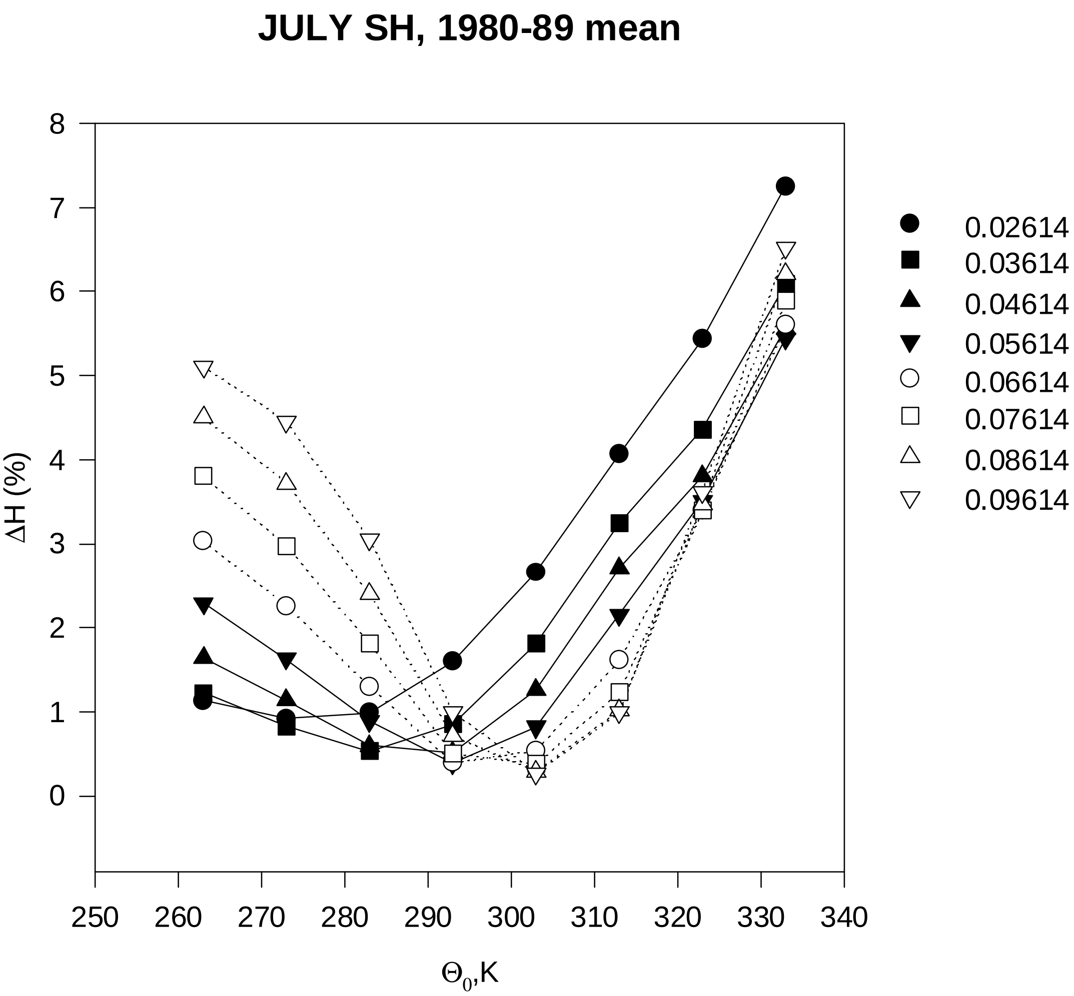

Figure 1. Dependence of informational entropy deficit ΔH on the potential temperature $\Theta = \Theta_0$ at the maximum of $\left| dp^*(\Theta)/d\Theta \right|$ in (2) for 8 different values of the parameter C (K^{-1}) that determines the vertical temperature lapse rate. The case of the Southern Hemisphere, July is considered; ΔH are calculated using yearly monthly-mean statistics averaged over 1980-89 period, computed ΔH values are expressed in percents of $\ln 19$.

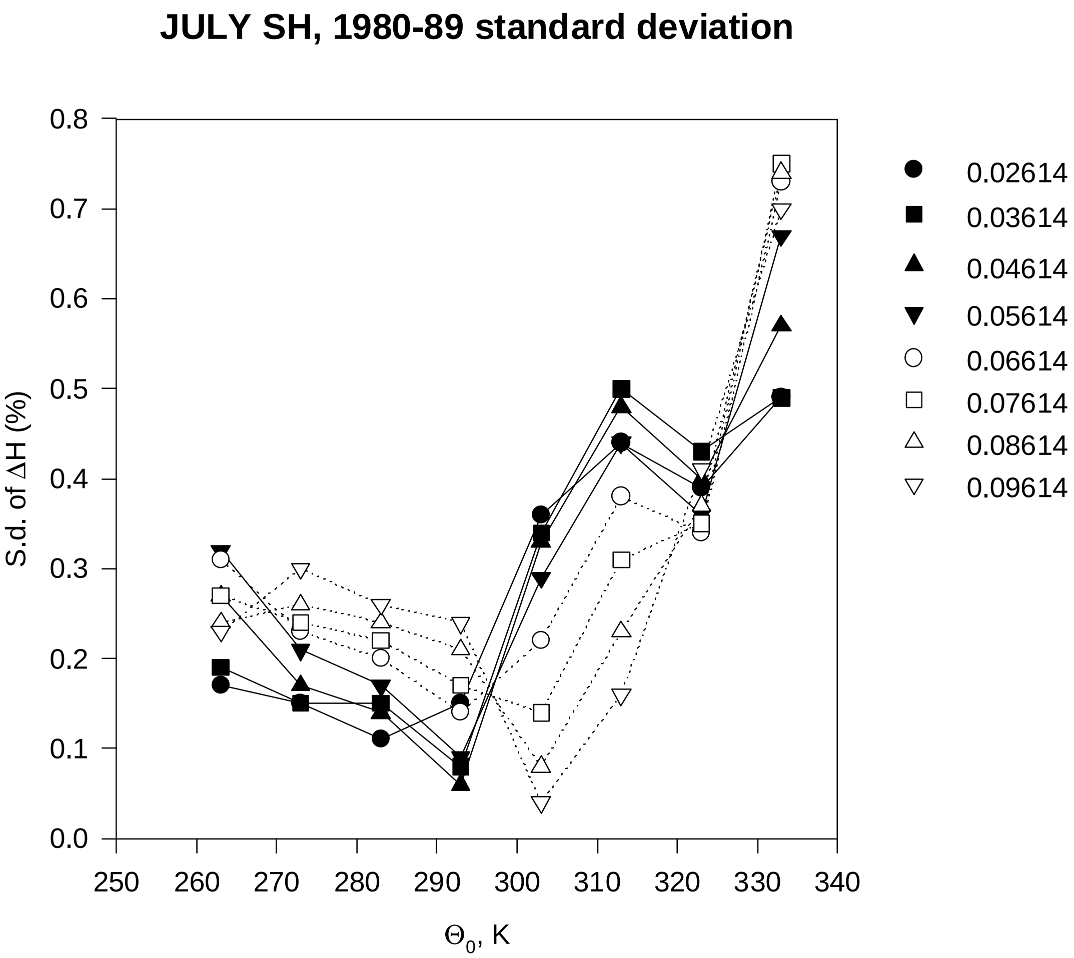

Figure 2. Standard deviations of ΔH from their mean values for 1980-89. Other notations are as in Fig. 1; SH, July.

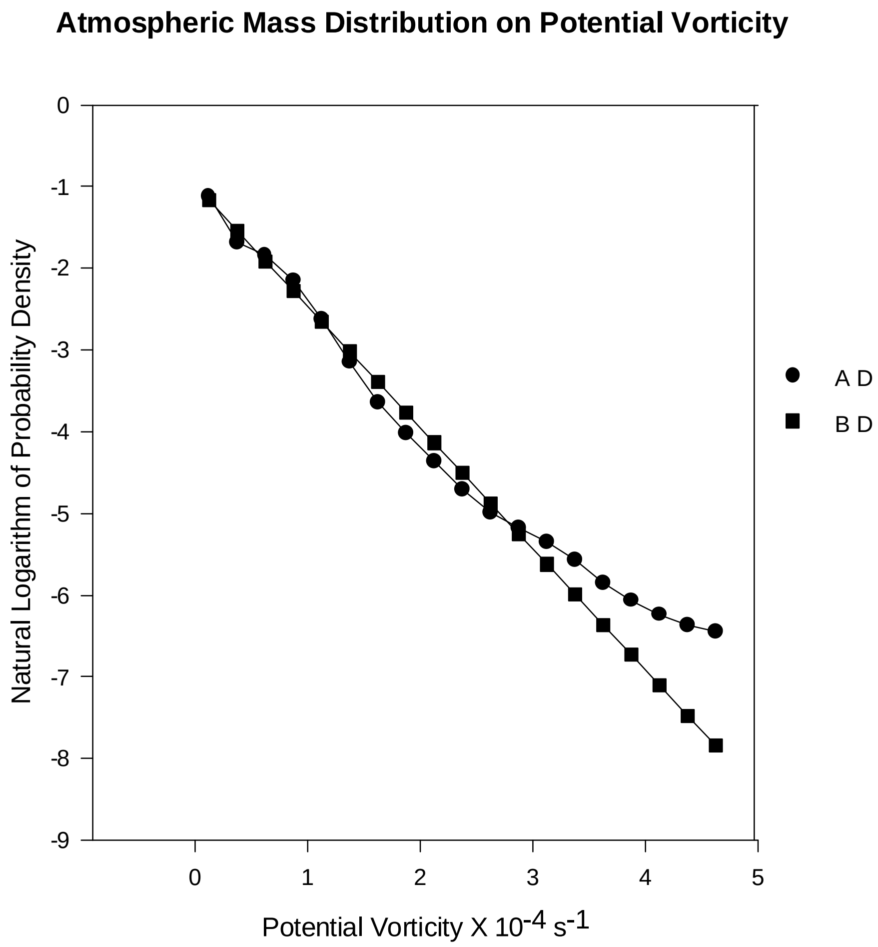

Figure 3. Atmospheric mass distribution on optimally modified PV (Θ_0=293 K, C=0.04614 K^{-1}) for NH, January 1980; AD is actual distribution and BD is corresponding the reference distribution.

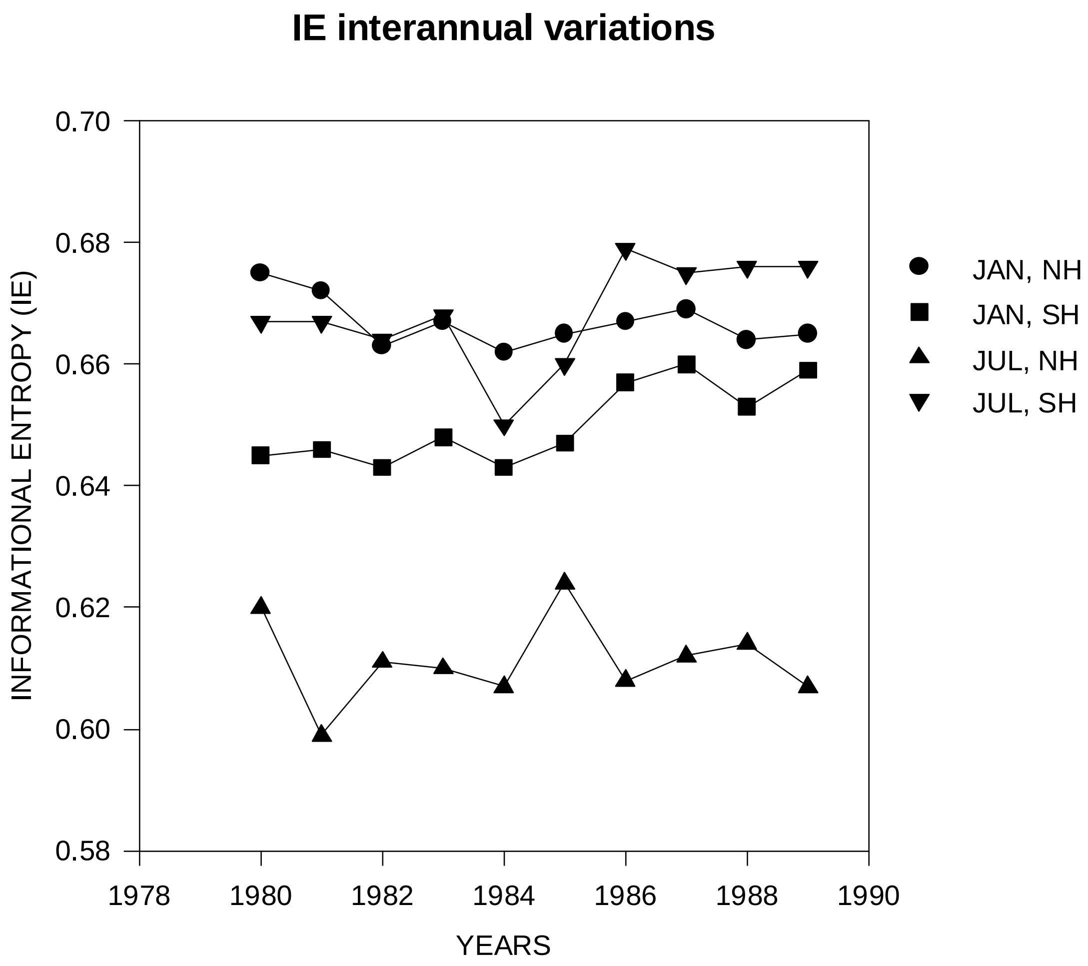

Figure 4 Interannual variability of the informational entropy H of monthly-mean air mass distribution on optimally modified PV for 1980-89 period.

After the subtraction of the linear trends the correlation matrix $r'_{ij} = r\left(Y_i - \overline{Y}_i, Y_j - \overline{Y}_j\right)$ becomes

$$\mathbf{r}' = \begin{pmatrix} 1 & 0.83^* & 0.04 & 0.68^* \\ 0.83^* & 1 & -0.20 & 0.81^* \\ 0.04 & -0.20 & 1 & -0.14 \\ 0.68^* & 0.81^* & -0.14 & 1 \end{pmatrix}.$$

We observe that the hemispheres in July are discoupled but in January they are strongly enough linked in the interannual time-scale. Possible linkage might originate from either phenomena like El Nino events occurring around January or standing planetary waves propagation across the equator. Due to huge thermal inertia of oceans dominating over the Southern Hemisphere, Y_2 and Y_4 values are well-correlated both in decadal and interannual time scales. Linkage between Y_1 and Y_4 occurs via Y_2, and $r'_{14} \approx r'_{12} r'_{24}$.

Using arguments based on the impermeability property of isentropic surfaces for the “potential vorticity substance” (Haynes and McIntyre 1990), it can be shown that the hemispheric value of |Q| becomes a monotonic decreasing function of the polar surface air potential temperature if three conditions hold: (i) equatorial surface air potential temperature is constant, (ii) surface air potential temperature decreases monotonously in equator-to-pole direction, and (iii) isentropic distribution of the “vertical” component of absolute vorticity ζ_Θ (see, Section 2) is kept constant. Because the surface air pressure varies much less than the surface air temperature does, it follows that |Q| is a monotonic increasing function of the equator-to-pole surface air temperature difference ΔT_s, provided that the isentropic distribution of ζ_Θ is fixed: $\left[\partial |Q| / \partial (\Delta T_s)\right]_{\zeta_\Theta = const} > 0$. In a general case, the changes in |Q|, accompanying those of ΔT_s, are given by the formula $d|Q| = \left[\partial |Q| / \partial (\Delta T_s)\right]_{\zeta_\Theta = onst} d(\Delta T_s) + \delta |Q|$,

where an appearing additional term δ|Q| stands for possible changes in ζ_Θ distribution. Nevertheless, using the thermal wind equation it could be shown that the δ|Q| contribution to the d|Q| value is 10-100 times smaller than the first term value. In particular, the resulting formula predicts that seasonal variations in |Q| values are to be well correlated with those in ΔT_s. This is clearly seen in data presented in Table 1 which depicts higher values of |Q| in corresponding winter months for both hemispheres.

Actual H values were approximated by $H_B \approx (\ln 19)^{-1} \ln|Q| + const$ and the linear regression equations used. From the formulae written above it follows that $\eta_i = \left|Q_i^{1989}\right| / \left|Q_i^{1980}\right| = 19^{10 a_i}$, where a_i are the linear regression coefficients and the indices i=1,2,3,4 have the same meaning as before. The data in Table 2 show, that $\eta_1=0.980$ (Northern Hemisphere, January), $\eta_2=1.053$ (Southern Hemisphere, January), and $\eta_4=1.044$ (Southern Hemisphere, July). The vortex charge trend for Arctic in July is negligible. Very similar results could be obtained if a linear regression was applied directly to the specific, per unit mass, vortex charge values Q given in Table 1.

This means that in the 1980s one finds gradual cooling over Antarctic for both seasons and warming over Arctic in January. As concerns the ΔT_s trends, our results are only qualitative. The remaining problem is to give a quantitative estimate of these trends, but one needs more knowledge of the functional dependence of |Q| on ΔT_s. Preliminary analysis of direct observational data confirms, as a whole, temperature increase over Arctic and temperature decrease over Antarctic in the 1980s. If the composite, i.e. averaged over January and July, values of both equatorial and mean hemispheric surface air temperature are used, then, based on some natural assumptions for the meridional temperature profile, it can be deduced that during this decade the equator-to-pole surface temperature difference in the Northern Hemisphere decreased by 0.3 K and in the Southern Hemisphere increased by a little bit more than 0.1 K.

CONCLUDING REMARKS

(1) By using the existing arbitrariness ("gauge invariance") in a general Ertel's PV definition it is possible to construct a PV modification q_0 which corresponds to the highest possible degree of closeness between actual and reference PV distributions, the latter having the same amount (per unit mass) of vortex charge Q over the Hemisphere and the maximum value of the informational entropy. Optimal choice of an arbitrary function of potential temperature $\chi_0(\Theta)=-p^*_0(\Theta)/g$, entering q_0, results in the $\mu(q_0,\chi_0)$ distribution, which is similar to an idealised statistically equilibrium distribution $\mu_B(q_0,\chi_0)$, having no dependence on χ_0, i.e. with the equipartition of atmospheric mass between equal intervals of χ_0.
(2) When inspecting the January and July PV statistics temporal behaviour we detected a progressive growth of the "vortex-charge-equivalent" equator-to-pole air surface temperature difference in the Southern Hemisphere during the 1980s. This trend is accompanied by a decrease of the "vortex-charge-equivalent" meridional temperature gradient in January for the Northern Hemisphere. It is worth mentioning that according to Kelly and Jones (1996), who analysed monthly-mean surface air temperature data for 1959-1993 period, the temperature change over Antarctica and surrounding regions, though weak, is of opposite sense to that of the global as a whole.

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