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1. INTRODUCTION*

It can be shown that the freedom available in Ertel's potential vorticity definition allows one to arrive at a pair of $(q, \chi(\Theta))$ -variables [q is the modified potential vorticity (MPV), $\chi(\Theta)$ is the appropriately taken function of potential temperature Θ which enters MPV-definition], such that the air mass enclosed within solenoids formed by intersection between the families of equiscalar surfaces $q=\text{const}$ and $\chi=\text{const}$ is nearly preserved despite the influence of diabatic heating and turbulent friction. It is hypothesized and attempted to prove that in (q, χ) -co-ordinates the long-term atmospheric climate processes admit a simple statistical description.

2. BASIC STATISTICAL ARGUMENTS

Let the quantity $\mu(q, \chi)dq d\chi$ depicts a portion of the total atmospheric mass enclosed within the (q, χ) -solenoid, with cross-sectional area equal to $dq d\chi$ [in (q, χ) -phase space]. If $\iint \mu(q, \chi)dq d\chi=1$, where integration is taken over all q and χ values, then $\mu(q, \chi)$ -function may be regarded as the 2D probability density of q and χ values for a randomly chosen air particle (Obukhov, 1964). When being displayed in (q, χ) -co-ordinates, the atmosphere is close to an idealized reference state (Kurgansky & Pisnichenko, 2000). This reference state, defined for the two hemispheres separately, is described by a probability density μ_B depending on q solely:

$$\mu_B(q)=|Q|^{-1} \cdot \exp(-q/Q).$$

Here, $Q=\int q \mu(q, \chi)dq d\chi$ is the first moment of $\mu(q, \chi)$, named hereafter the atmospheric vortex charge. Further on, $\mu(q, \chi)$ is reduced to 1D probability density $\mu(q)=\int \mu(q, \chi)d\chi$ by integrating over all χ . The reference $\mu_B(q)$ -distribution supplies the maximum of the informational entropy $H=-c \int \mu \ln \mu dq$ (c is a normalized coefficient), provided $Q=\text{const}$. The informational entropy deficit $\Delta H=H_B-H$ ($H_B=-c \int \mu_B \ln \mu_B dq=c \ln|Q|+\text{const}$) characterizes the degree of closeness between $\mu(q)$ and $\mu_B(q)$ distributions. Moreover, it gives a tool how minimizing ΔH for different χ -functions to arrive at the best choice of latter resulting in the optimally modified PV.

3. INFORMATIONAL ENTROPY INTERANNUAL VARIATIONS

Temporal behavior of informational entropy H within 1980-89 period was studied for two hemispheres, separately. Global ECMWF data for January and July

were used. Using the procedure of minimizing of ΔH , the optimal χ -function and the corresponding potential vorticity modification q were found. To do it, the "trial functions"

$$\chi \Theta = A \arctan C \Theta - \Theta_0 - \pi/2$$

were used, which correspond to Cauchy distribution in statistics. Here, A , C and Θ_0 are variable parameters and only two of them, i.e. C and Θ_0 , could be chosen independently because of the total atmospheric mass constancy constraint. It was found for both months and both hemispheres $\Delta H=\Delta H(C, \Theta_0)$ attains its minimum values in the close vicinity of a line given by the linear regression equation $\Theta_0-292.55=321.4 \times (C-0.04614)$. For a further analysis, the values $\Theta_0=293$ K and $C=0.04614$ K⁻¹, lying very close to the regression line, were chosen. Interannual changes in the informational entropy H of monthly-mean $\mu(q)$ -distribution for 1980-89 period are presented in Figure 1.

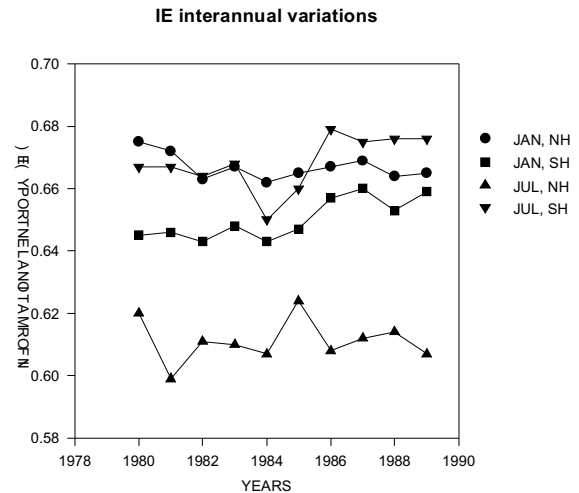


Figure 1

We denote the H values for different months and hemispheres as $Y_1=H_{JAN}^{NH}$, $Y_2=H_{JAN}^{SH}$, $Y_3=H_{JUL}^{NH}$, $Y_4=H_{JUL}^{SH}$. Correlation matrix $r_{ij}=r(Y_i, Y_j)$, $i, j=1, 2, 3, 4$, calculated on the basis of data used to plot Figure 1, is given by

$$r = \begin{pmatrix} 1 & 0.09 & 0.03 & 0.26 \\ 0.09 & 1 & -0.11 & 0.82 \\ 0.03 & -0.11 & 1 & -0.11 \\ 0.26 & 0.82 & -0.11 & 1 \end{pmatrix}$$

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where the only numbers marked by asterisks are statistically significant. Figure 1 clearly demonstrates similar behavior of Y_1 and Y_2 curves but the corresponding correlation coefficient is negligible. This originates from essential linear trends incorporated in informational entropy values. To eliminate this linear trend, the linear regression equations $Y_i = a_i X + b_i$, $i=1,2,3,4$, where X denotes years and varies in the range of 80-89, were obtained for every Y_i . After the subtraction of the linear trends the correlation matrix $r'_{ij} = r(Y_i - \bar{Y}_i, Y_j - \bar{Y}_j)$ becomes

$$r' = \begin{pmatrix} 1 & 0.83^* & 0.04 & 0.68^* \\ 0.83^* & 1 & -0.20 & 0.81^* \\ 0.04 & -0.20 & 1 & -0.14 \\ 0.68^* & 0.81^* & -0.14 & 1 \end{pmatrix}.$$

We observe that the hemispheres in July are decoupled but in January they are strongly linked in the interannual time-scale. Possible linkage might come from either phenomena like El Nino events occurring around January or standing planetary waves propagation across the equator. Due to huge thermal inertia of oceans dominating over the Southern Hemisphere, Y_2 and Y_4 values are well correlated both in decadal and interannual time scales. Linkage between Y_1 and Y_4 occurs via Y_2 , and it seems to be not accidental that $r'_{14} \approx r'_{12} r'_{24}$.

Using arguments based on the isentropic surfaces impermeability property for the "potential vorticity substance" (Haynes & McIntyre, 1990), it can be shown that the hemispheric value of $|Q|$ becomes the monotonic decreasing function of a surface air potential temperature Θ over the geographic pole if three conditions hold: (i) equatorial surface air Θ -value is a prescribed constant, (ii) surface air Θ -values decrease monotonously in equator-to-pole direction, and (iii) isentropic distribution of the "vertical" component of absolute vorticity ζ_Θ is kept constant. Because of surface air pressure varies much less than surface air temperature does, it follows that $|Q|$ is a monotonic increasing function of the equator-to-pole surface air temperature difference ΔT_s , provided isentropic distribution of ζ_Θ is fixed: $\alpha = \left. \frac{d|Q|}{d \Delta T_s} \right|_{\zeta_\Theta = \text{const}} > 0$. The

changes in $|Q|$, accompanying those of ΔT_s , are given by the formula $d|Q| \approx \alpha d(\Delta T_s)$. In particular, this formula predicts that seasonal variations in $|Q|$ should be well correlated with those in ΔT_s . This is clearly seen in Figure 1 which depicts higher values of H , which is proportional to $|Q|$, in corresponding winter months for both hemispheres.

Actual H values were approximated by $H_B \approx \ln|Q| / \ln 19 + \text{const}$. From the linear regression equations written above it follows that

$$\eta_i = \left| \frac{Q_i^{1989}}{Q_i^{1980}} \right| = 19^{10a_i}, \text{ where } a_i \text{ are the linear}$$

regression coefficients and indices $i=1,2,3,4$ have the same meaning as before. The calculations gave that $\eta_1=0.980$ (NH, January), $\eta_2=1.053$ (SH, January), and $\eta_4=1.044$ (SH, July). Vortex charge Q trend for Arctic in

July is negligible.

It means that in the 1980s one finds gradual cooling over Antarctic for both seasons and warming over Arctic in January. As concerns ΔT_s trends, our results are only qualitative. To give a quantitative estimate of these trends one needs more knowledge on the functional dependence of $|Q|$ upon ΔT_s . Preliminary analysis of direct observational data confirms, as a whole, the temperature increase over Arctic and its decrease over Antarctic in the 1980s. If the composite, i.e. averaged over January and July, values of both equatorial and mean hemispheric surface air temperature are used, then, basing on some natural assumptions on meridional temperature profile, it can be deduced that during this decade the equator-to-pole surface temperature difference in the NH decreased by 0.3 K and in the SH increased by a little bit more than 0.1 K.

Thus, when inspecting the temporal behavior of the January and July MPV statistics we detected a hint on the progressive growth of the equator-to-pole air surface temperature difference in the SH during the 1980s which is accompanied by a more pronounced decrease in the meridional temperature gradient in January over the NH. Attributing the latter to the overall warming in the NH one might hypothesize that the diminishing of the meridional temperature gradient should result in reduction of the standing planetary waves generation over the NH. This should lead to less wave-activity propagation into the SH, then to weakening of the induced meridional circulation there and, as a result, to a temperature decrease over the Antarctic polar region due to smaller adiabatic heating in the descending branch of the circulation in question. In this connection, it is worth to mention that according to Kelly and Jones (1996), who analyzed monthly-mean surface air temperature data for 1959-1993 period, the temperature change over Antarctica and surrounding regions, though weak, is in the opposite sense to that of the global as a whole.

To judge to which extent the discovered trends in $|Q|$ are meaningful, one has to extend MPV calculations onto the broader period covering the 1990s and to use alternative NCEP data, as well.

REFERENCES

- Haynes, P.H., and M.E.McIntyre, 1990: On the conservation and impermeability theorems for potential vorticity, *J.Atmos.Sci.*, **47**, 2021-2031.
 Kelly, P.M., and P.D.Jones, 1996: Removal of the El Nino - Southern Oscillation signal from the gridded surface air temperature data set, *Journ.Geophys.Res.*, **101**, No D14, 19,013-19,022
 Kurgansky, M.V., I.A. Pishnichenko, 2000: Modified Ertel's potential vorticity as climate variable, *J.Atmos.Sc.*, **57**.
 Obukhov, A.M., 1964: Adiabatic invariants of atmospheric processes, *Meteorologiya i Gidrologiya*, No.2, 3-9.